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Theoretical model of a DC magnetohydrodynamic generator in annular geometry

Modelo teórico de generador magnetohidrodinámico DC en geometría anular

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Abstract

We developed a theoretical model of a liquid metal magnetohydrodynamic (MHD) generator in annular geometry operating in direct current (DC) mode. The geometrical concept of the MHD generator consists of a very thin annular duct where the conducting fluid flows due to a constant pressure gradient in an imposed azimuthal magnetic field. We have supposed negligible effects of the induced magnetic field which is characteristic of MHD flows at very low magnetic Reynolds numbers. These assumptions reduce the MHD equations to one dimensional fully developed flow where the induced current is given only by Ohm's law. The theoretical performance of the generator is analyzed as a function of the external electrical load for different operating conditions. The electrical output power depends on the imposed magnetic field, the electrical conductivity of the fluid, its velocity and the external electrical load. The maximum output power occurs when the external resistance equals the internal resistance of the generator. We found that the internal resistance depends on the imposed magnetic field and geometrical parameters as in the case of the classical MHD generator in rectangular geometry, in spite of the absence of Hartmann layers. We analyze the isotropic electrical efficiency of the MHD generator for an external electrical load ranging from negligible resistance (short circuit) to very large resistance (open circuit) conditions. For a given external load the higher efficiencies of the generator can be achieved by increasing the imposed magnetic field.

Resumen

Desarrollamos un modelo teórico de generador magnetohidrodinámico (MHD) de metal líquido en geometría anular que opera en modo de corriente directa (DC). El concepto geométrico del generador MHD consiste de un ducto anular muy delgado por donde fluye el fluido conductor debido a un gradiente de presión constante en un campo magnético azimutal impuesto. En este modelo se consideró la aproximación de campo magnético inducido depreciable, que es característico de los flujos MHD a muy bajos números de Reynolds magnéticos. Estas suposiciones reducen las ecuaciones de MHD a un flujo unidimensional completamente desarrollado donde la corriente inducida viene dada solo por la ley de Ohm. Se analizó la eficiencia teórica del generador y la potencia eléctrica de salida en función de la carga eléctrica externa para diferentes condiciones de operación. Se encontró que la potencia máxima entregada por el sistema se produce cuando la resistencia externa es igual a la resistencia interna del generador. Sin embargo, se encontró que la resistencia interna se comporta similar a la del generador MHD en geometría rectangular a pesar de la ausencia de capas límite de Hartmann. Analizamos la eficiencia eléctrica isotrópica del generador MHD para una carga eléctrica externa que varía desde una resistencia eléctrica despreciable (cortocircuito) hasta condiciones de alta resistencia (circuito abierto). Para una carga externa dada, la eficiencia del generador se incrementa conforme se incrementa la intensidad del campo magnético impuesto.

Keywords:
Magnetohydrodynamic generator; energy conversion systems; linear generator

Palabras clave:
Generador Magnetohidrodinámico, sistemas de conversión de energía, generador lineal

Introduction

An MHD generator converts the motion of an electrically conducting fluid directly into electrical energy through the interaction of the fluid in motion with an applied magnetic field. The liquid metal MHD generator uses simple geometry in its mechanical design and could be more efficient than the conventional generation systems due to the absence of turbines or prime motors. The most simplified conceptualization of a typical MHD generator consists of three basics elements: 1) a rectangular cross-section duct, where the liquid metal flows, 2) the electrodes, responsible for collecting the induced currents in the generator and forming the lateral walls of the duct and 3) a magnetic field source. Thus, when the conducting fluid moves in the duct, it interacts with the imposed magnetic field inducing an electric current perpendicular to the movement of the fluid and to the applied magnetic field. The induced electrical current can be extracted through an external circuit (electrical load) connected to the electrodes. If the motion is unidirectional then the device works as a direct current (DC) generator.

Traditionally, theoretical models of MHD generators have been divided according to the physical characteristics of the fluid. The ideal case considers an inviscid fluid, which we denote the Faraday MHD generator, while other considers a viscous fluid, which we denote the Hartmann MHD generator. In the past, several theoretical models of MHD generators in a rectangular duct geometry have been presented. Jackson [1], presents the model of a direct current MHD Faraday generator, calculating the electrical power output and the efficiency of the system, and expresses the internal resistance of the machine in terms of the dimensions of the MHD channel and the electrical conductivity of the fluid. Hughes et al [2], show an equivalent circuit of a direct current MHD Hartmann generator and calculate its internal resistance using the electrical characteristics of the generator. The MHD generator configuration is generally in the geometry of a rectangular channel and some experimental prototypes have been designed using MHD generator models in rectangular channel operating in DC mode [3-7]. However, the interest in studying other geometries is still present.

In this paper, we study a liquid metal MHD generator when the fluid is driven by a constant pressure gradient, in an annular duct. In our model, we consider the presence of a purely azimuthal magnetic field in the annular region, where the electrodes are the walls of the annular duct formed by two coaxial cylinders. We assume that the azimuthal magnetic field, can be induced by a cylindrical superconductor. The use of superconductors in the construction of electromagnets in MHD generators has been previously proposed [8-9-10]. The induction of azimuthal magnetic field by a superconductor filament was demonstrated by [11].

The MHD generators in annular duct geometry, as well as the behavior of the MHD flow have been studied in the last century. In fact, analytical solutions for the MHD laminar flow in an annular duct under a purely radial magnetic field, are well known [12-13-14-15]. Elco et al. [12], proposed an MHD system of direct current generation, formed by two parallel discs located axially at a fixed distance of separation. These discs are the electrodes of the generator. In the space between the electrodes, an annular duct is formed using two cylindrical magnets, one placed in the center and the other in the outer radius. The magnets are radially magnetized, so that in the annular region a radial magnetic field exists. The authors calculated the electrical and thermal performance of the system. On the other hand, Rao and Erteza [16], studied a multipolar DC generator, its annular section being constructed by two coaxial cylinders where an ionized gas flows. The magnetic poles and electrodes are placed on the perimeter of the internal and external cylinder. The distribution of the magnetic poles is such that, in the annular region, the magnetic field component above the magnetic pole is radial and azimuthal on the electrodes, forming essentially rectangular MHD channels distributed on the annular section. In their analysis, the authors found the internal impedance of the generator, in the same way as Hughes [2].

In comparison with previous models, in the present contribution we extend the previous theoretical solution, analyze the generator performance in terms of electric power as a function of the electric load and find the theoretical efficiency of the generator, using a variation of the Hartmann model and considering the inductionless approximation [17], together with the approximation of the small annular space [18-19].

General formulation

The proposed MHD generator of annular geometry is shown in fig. 1. It consists of two concentric cylinders of radial lengths r_i and r_o, forming a very thin annular duct, which has a thickness δ, where an incompressible conducting fluid flows axially in steady-state, in an azimuthal magnetic field. The length of the MHD duct is L and is greater than δ so that the edge effects at the entrance and exit of the MHD generator can be neglected. It is also assumed that the imposed magnetic field is independent of the axial coordinate. The interaction between the fluid in motion and the magnetic field induces a radial electric current that can be extracted through the electrodes to an external circuit. The walls of the annular duct formed by the inner and outer cylinders constitute the electrodes of the generator. The flow dynamics is governed by the Navier-Stokes equations with the Lorentz force as the body force which results from the interaction between the induced current and the azimuthal magnetic field.

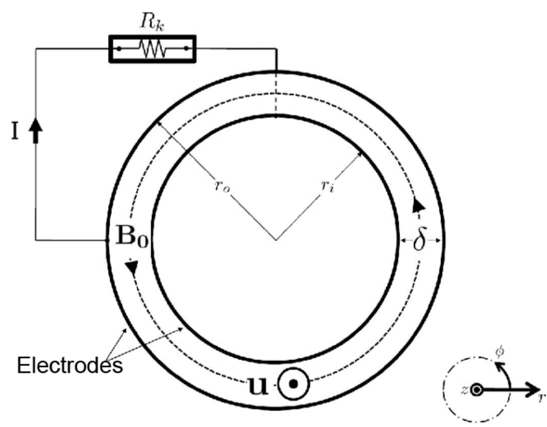

Figure 1. Sketch of the cross-section of an MHD Generator in annular geometry.

Formulation of problem

The flow in the generator is purely axial, axisymmetric and fully developed so that the velocity profile has the form, $\boldsymbol{u}=u_z(r)\boldsymbol{e}_z$ where $\boldsymbol{e}_z$ is the axial unit vector. The imposed magnetic field is given by $\boldsymbol{B}_0=(S/r)\boldsymbol{e}_\phi$, where S measures the field strength and $\boldsymbol{e}_\phi$ is the azimuthal unit vector in a cylindrical polar coordinate system (r,ϕ,z). Since the fluid motion just perturbs slightly the applied field $\boldsymbol{B}_0$, the induced magnetic field, $\boldsymbol{b}$, associated with the induced currents in the generator ($\sim\sigma\, u\times\boldsymbol{B}_0$) is negligible with respect to $\boldsymbol{B}_0$. This leads to the so called inductionless approximation [17] which is characteristic of MHD flows at very small magnetic Reynolds number $R_m=\mu\sigma u_0 l$, where μ is the magnetic permeability, σ the electrical conductivity, and u_0 and l are the characteristic velocity and length scales of the flow, respectively.

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Electromagnetic formulation

Induced current and voltage

The induced current $\boldsymbol{J}$ in the transducer is given by the Ohm's law for a moving media as

$$\boldsymbol{J} = \sigma\left(\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B}_0\right) \tag{1}$$

where $\boldsymbol{E}$ is the electric field associated to the external electrical load. In the present case, the Ohm's law reduces to

$$j_r(r) = \sigma\left(E_r - \frac{u_z(r)S}{r}\right) \tag{2}$$

Here E_r and j_r are the radial component of the induced electric field and current density, respectively. Due to the inductionless approximation, $\boldsymbol{E}$ could be considered as a conservative field, so that $\nabla\times\boldsymbol{E}\approx 0$ and $\boldsymbol{E}=-\nabla\varphi$, φ being the electrostatic potential. Moreover, charge conservation requires $\nabla\cdot\boldsymbol{J}=0$, implying that $j_r \sim 1/r$, whose constant of proportionality for convenience is defined as $-\sigma c$. Here, c is some constant which depends on the external circuit conditions and it will be determined later. Inserting j_r in (2) and considering the previous assumptions we obtain

$$j_r = -\sigma\left(\frac{\partial\varphi}{\partial r} + \frac{u_z(r)S}{r}\right) \tag{3}$$

We calculate the total current I flowing through the electrodes as

$$I = \int_S \boldsymbol{J}\cdot\boldsymbol{n}\, ds = -\int_0^{2\pi}\int_0^{L} j_r r dz d\phi = 2\pi L\sigma c, \tag{4}$$

and the total induced voltage V_T between the terminals as [2],

$$V_T = -\oint \boldsymbol{E}\cdot d\boldsymbol{l} = \int_{r_i}^{r_0} E_r dr = -\int_{r_i}^{r_0}\frac{\partial\varphi}{\partial r}dr = -\Delta\varphi \tag{5}$$

If an external electrical load R_k is connected between the terminals of the generator, the total voltage across the load is just

$$V_T = R_k I \tag{6}$$

Additionally, in the equivalent circuit [2], the voltage V_T is represented as the sum of the open circuit voltage V_{oc} plus the voltage drop due to the internal resistance of the generator R_i, this relation is written as

$$V_T = -R_i I + V_{OC} \tag{7}$$

Total electric power

The output electric power, P_e, is calculated from the relation $P_e = V_T\, I$ or from the definition

$$P_e = -\int_v \boldsymbol{J}\cdot\boldsymbol{E}\, dv = 2\pi L\sigma c\int_{r_i}^{r_0} E_r dr \tag{8}$$

where v is the volume of the generator.

Hydrodynamic formulation

The fluid velocity $\boldsymbol{u}$ satisfies the incompressibility condition $\nabla\cdot\boldsymbol{u}=0$, and is governed by the Navier-Stokes equation

$$\frac{\partial\boldsymbol{u}}{\partial t} + \left(\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{J}\times\boldsymbol{B}_0 \tag{9}$$

where the term $\boldsymbol{J}\times\boldsymbol{B}_0$ is the Lorentz force. Considering the assumptions given in Section 2 and Section 3, eq. (9) simplifies as

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) - \frac{\sigma c S}{\rho r^2} = 0 \tag{10}$$

where p is the pressure, ν kinematic viscosity and ρ the mass density.

The mechanical driven power is related to the balance between the pressure gradient and the Lorentz force, in this sense we define the flow power as

$$P_f = -\int_v \left(\boldsymbol{J}\times\boldsymbol{B}_0\right)\cdot\boldsymbol{u}dv = 2\pi LS\sigma c\int_{r_i}^{r_0}\frac{u_z}{r}dr \tag{11}$$

Physically, it is the mechanical power needed to overcome the Lorentz force in order to produce useful electricity.

The small gap approximation

In the following theoretical development, we assume a very small annular space defined by $r_i \gg \delta$ leading to negligible curvature effects. Firstly, we define the reduced variable χ as $\chi=(r-r_i)/\delta$ where $\chi\in[0,1]$. This approximation reduces the charge conservation condition to $j_r\approx(-\sigma c)/r_i$ and the Ohm's law (2) in terms of χ to

$$j_r \approx -\sigma\left(\frac{1}{\delta}\frac{\partial\varphi}{\partial\chi} + \frac{u_z S}{r_i}\right) \tag{12}$$

Applying this approximation to the momentum equation (10) in terms of χ, it reduces to

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$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\nu}{\delta^2}\frac{\partial^2 u_z}{\partial \chi^2}-\frac{\sigma c S}{\rho r_1^2}=0 \tag{13}$$

In the Appendix A, we describe in more detail the small gap approximation.

Electrical efficiency of the MHD generator

The isotropic electrical efficiency of the MHD generator can be defined as the ratio of the output electrical power to the flow power, that is

$$\eta=\frac{P_e}{P_f} \tag{14}$$

Direct Current model

In the DC model, the fluid motion is driven by a constant pressure gradient of the form $-dp/dz=G$, producing a steady velocity in the z axial direction, which in terms of the reduced variable χ reads $u=u_z(\chi)\mathbf{e}_z$ Then, the fluid motion described by equation (13) simplifies as

$$\frac{G}{\rho}+\frac{\nu}{\delta^2}\frac{\partial^2 u_z}{\partial \chi^2}-\frac{\sigma c S}{\rho r_i^2}=0 \tag{15}$$

Equation (15) written in terms of the dimensionless variables $\hat{G}=G\delta^2/\rho\nu u_0$, $\hat{u}_z=u_z/u_0$, $\hat{r}_i=r_i/\delta$, $\hat{c}=c/S\, u_0$ and $Ha=\sqrt{\sigma/\rho\nu}$ becomes

$$\frac{\partial^2 \hat{u}_z}{\partial \chi^2}-\frac{\hat{c}Ha^2}{\hat{r}_i^2}+\hat{G}=0 \tag{16}$$

Where u_0 is the hydrodynamic mean velocity and Ha is the Hartmann number which estimates the ratio of the Lorentz to viscous forces. The analytical solution of the last equation that satisfies non-slip boundaries conditions at the inner and outer walls is

$$\hat{u}_z=\left(\frac{\hat{G}}{2}-\frac{\hat{c}Ha^2}{\hat{r}_i^2}\right)(1-\chi)\chi \tag{17}$$

The average velocity $u_m=\int_0^1 u_z d\chi$ expressed in dimensionless quantities is

$$\hat{u}_m=\frac{1}{12}\left(\hat{G}-\frac{\hat{c}Ha^2}{\hat{r}_i^2}\right) \tag{18}$$

where $\hat{u}_m=u_m/u_0$.

In the following we determine the electrical characteristics of the MHD generator. The induced current in the generator, given by integrating eq. (12) in the interval [0,1], results as

$$j_r=-\sigma u_m\frac{S}{r_i}(1-\boldsymbol{K}) \tag{19}$$

where $K=-\Delta\varphi r_i/\delta u_m S$ is the load factor parameter which depends on the electrical conditions at the external circuit. For short-circuit condition K is zero, while for open-circuit condition, K is equal to unity. If we replace charge conservation equation $j_r\approx-\sigma c/r_i$ into eq.(12) and we integrate it from zero to unity, the resulting equation is

$$\frac{c}{r_i}=-\frac{1}{\delta}V_T+u_m\frac{S}{r_i} \tag{20}$$

In dimensionless terms (20) reads as

$$\hat{c}=-\frac{\hat{r}_i}{Ha}\hat{V}_T+\hat{u}_m \tag{21}$$

where $\hat{V}_T=V_T/u_0\sqrt{\rho\nu/\sigma}$. Inserting (18) in (21) and solving for the constant $\hat{c}$, we get

$$\hat{c}=\frac{\hat{r}_i^2\left(Ha\hat{G}-12\hat{V}_T\hat{r}_i\right)}{Ha\left(Ha^2+12\hat{r}_i^2\right)} \tag{22}$$

The dimensionless total current (4) flowing through the electrodes is

$$\hat{I}=2\pi\hat{c}Ha \tag{23}$$

where $\hat{I}=I/Lu_0\sqrt{\rho\nu\sigma}$. Inserting (22) in (23), and solving for $\hat{V}_T$ we obtain the dimensionless terminal voltage as

$$\hat{V}_T=-\frac{\hat{I}\left(Ha^2+12\hat{r}_i^2\right)}{24\pi\hat{r}_i^3}+\frac{Ha\hat{G}}{12\hat{r}_i} \tag{24}$$

For open circuit conditions $\hat{I}=0$, and the voltage is given as $\hat{V}_{oc}=Ha\,\hat{G}/(12\hat{r}_i)$. For short circuit conditions $\hat{V}_T=0$ and the current is $\hat{I}_{sc}=(2\pi\,\hat{r}_i^2 Ha\hat{G})/(Ha^2+12\hat{r}_i^2)$. The dimensionless internal resistance $\hat{R}_i$ of the generator is given by ratio of the open circuit voltage to the short circuit current [2]. The resulting equation for $\hat{R}_i$ is

$$\hat{R}_i=\frac{Ha^2+12\hat{r}_i^2}{24\pi\hat{r}_i^3} \tag{25}$$

In fig.2 a) we show the dimensionless internal resistance of the generator as a function of Ha for different values of $\hat{r}_i$. In the plot we observe that $\hat{R}_i$ increases as the magnetic field increases, this behavior is more sensitive for $\hat{r}_i$<50. For the case of vanishing magnetic field $Ha\rightarrow 0$, the internal resistance reduces to the geometrical electrical resistance $\delta/2\pi\sigma L r_i$. In

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fig.2 b), for all cases the internal resistance decreases when $\hat{r}_i$ increases. For extremely large values of $\hat{r}_i$, the internal resistance tends to zero. These two graphs show that the internal impedance of the MHD generator is dependent on the magnetic field and the internal radius of the generator.

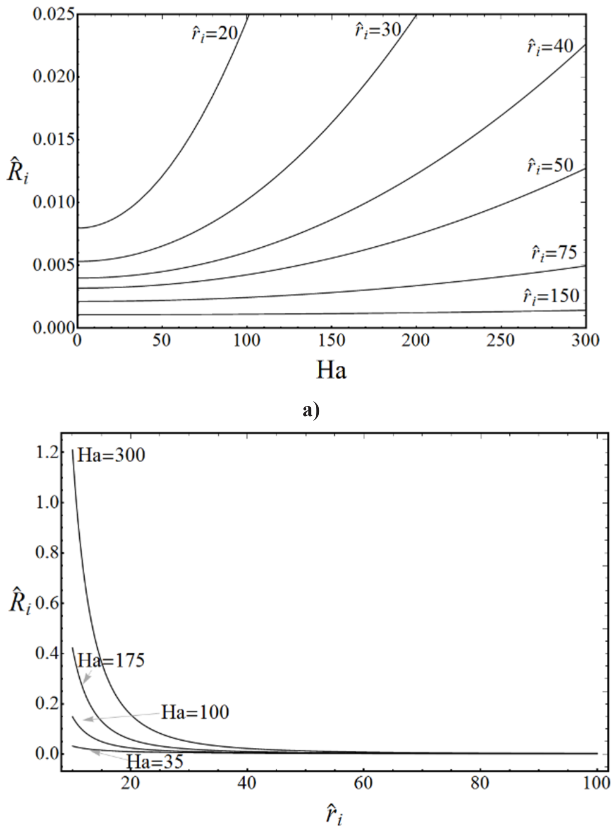

Figure 2. Dimensionless internal resistance $\hat{R}_i$ a) as a function of Ha for different values of $\hat{r}_i$ and b) as a function $\hat{r}_i$ for different values of Ha.

On other hand, if we express eq. (24) in terms of the internal resistance and the open circuit voltage, the resulting equation is

$$\hat{V}_T = -\hat{R}_i\hat{I} + \hat{V}_{0C} \tag{26}$$

Now, from eq. (6) the dimensionless voltage drop is given as $\hat{V}_T = \hat{I}\hat{R}_k$, where $\hat{R}_k = L\sigma R_k$. Inserting this relation in (26) and solving for the terminal voltage we obtain

$$\hat{V}_T = \left(\frac{\hat{R}_k}{\hat{R}_k + \hat{R}_i}\right)\hat{V}_{0C} = K\hat{V}_{0C} \tag{27}$$

where the load factor K is given in terms of load and internal resistances as

$$K = \frac{\hat{R}_k}{\hat{R}_k + \hat{R}_i} \tag{28}$$

Combining eqs. (26), (27) and (28), and solving for $\hat{I}$ we obtain

$$\hat{I} = \hat{V}_{0C}\frac{1-K}{\hat{R}_i} \tag{29}$$

The output electric power given in eq. (8) is expressed in dimensionless form by considering the results of eq. (27) and (29). The resulting equation is

$$\hat{P}_e = \frac{K(1-K)}{\hat{R}_i}\hat{V}_{0C}^2 \tag{30}$$

where $\hat{P}_e = P_e/L\rho\nu u_0^2$.

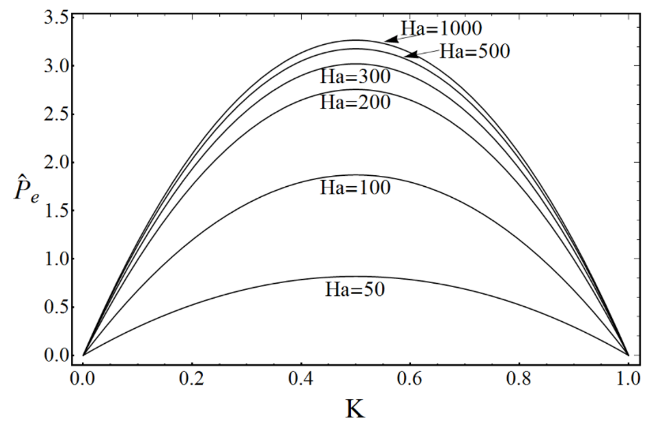

Figure 3. Dimensionless electric power $\hat{P}_e$ as a function of the load factor K for different values of Ha for a given $\hat{r}_i$.

In fig. 3, we plot the dimensionless electric power $\hat{P}_e$ as a function of the load factor for several values of Ha. Particularly, the maximum power $\hat{P}_{emax}$ occurs when the condition K=1/2 is satisfied, that is, when $\hat{R}_k \to \hat{R}_i$. In addition, we observe that the electric power increases as we increase the Ha values, however, when very large values of Ha are reached, the electric power does not increase in the same way as for small values of Ha. This behavior is also found for the maximum electrical power, $\hat{P}_{emax}$, as is shown in figure 4. In this case, $\hat{P}_{emax}$ reduces as

$$\hat{P}_{emax} = \frac{\hat{V}_{0C}\hat{I}_{SC}}{4} \tag{31}$$

In the graph is observed for given values of $\hat{r}_i$ how the maximum electrical power increases as Ha grows. In fact, from an asymptotic analysis to the equation (31), it was found that the maximum electrical power tends to a value of $\pi\hat{r}_i/24$.

Inserting (22) and (27) into (17) we obtain an expression for the velocity:

$$\hat{u}_z = \frac{\hat{G}}{2}\left(1 - \frac{Ha^2(1-K)}{24\pi\hat{r}_i^3\hat{R}_i}\right)(1-\chi)\chi \tag{32}$$

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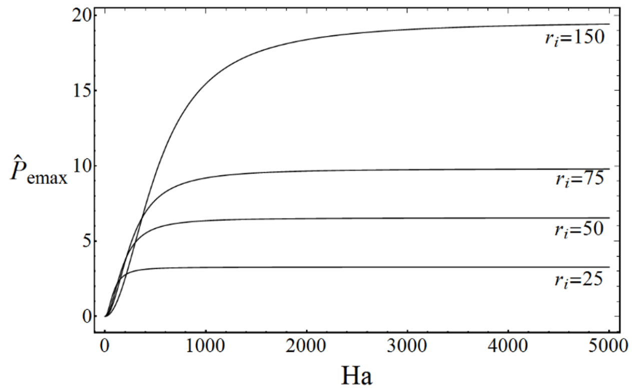

Figure 4. Maximum electric power $\hat{P}_{emax}$ as a function of *Ha* for some values of $\hat{r}_i$.

The average velocity in terms of *K* reduces to:

$$\hat{u}_m = \frac{\hat{G}}{12}\left(1-\frac{Ha^2(1-K)}{24\pi\hat{r}_i^3\hat{R}_i}\right) \tag{33}$$

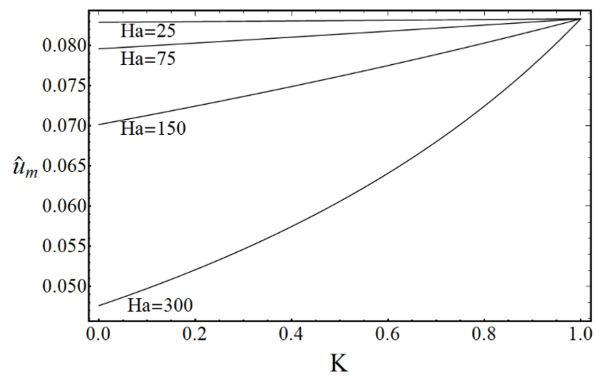

Figure 5. Average velocity $\hat{u}_m$ as a function of *K* for several *Ha*.

We plot in fig. 5 the average velocity as a function of *K*. Note that as *Ha* is increased, the flow slows down. This is because, as the magnetic forces become larger, the Lorentz force also increases and act in opposition to the fluid motion. For all *Ha*, the minimum value of average velocity is for short circuit condition, since the current in the circuit is maximum, and therefore, the Lorentz force is also maximum.

The flow power (11) in dimensionless form is:

$$\hat{P}_f = \frac{Ha(1-K)\hat{V}_{OC}^2\left(\frac{Ha^2(K-1)}{\pi\hat{r}_i^2\hat{R}_i}+\frac{24\hat{r}_i}{Ha}\right)}{24\hat{r}_i\hat{R}_i} \tag{34}$$

where $\hat{P}_f = P_f/(L\rho\nu u_0^2)$.

The fig. 6 shows the flow power as a function of *K* for several *Ha* . For short circuit condition the flow power is maximum and decreases as the load resistance increases, tending to zero, which corresponds to the open circuit condition.

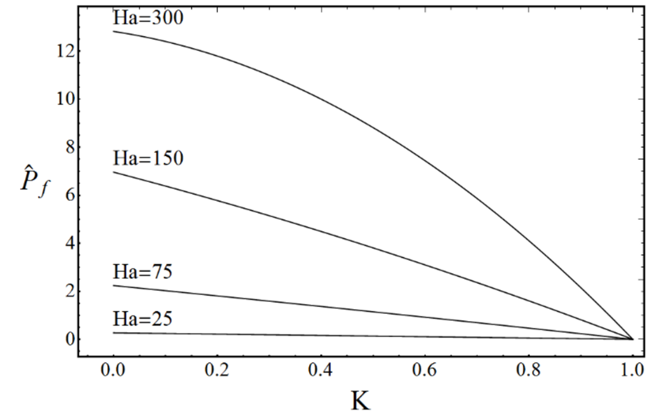

Figure 6. Dimensionless flow power $\hat{P}_f$ as a function of *K* for different *Ha*.

Finally, we calculated the electrical efficiency of the generator, defined in eq. (14), by using (30) and (34), the resulting equation is

$$\eta = \frac{K\left(Ha^2+12\hat{r}_i^2\right)}{Ha^2K+12\hat{r}_i^2} \tag{35}$$

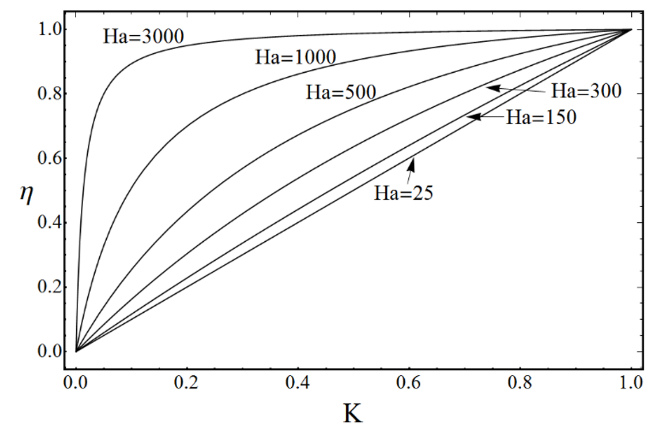

Figure 7. Electrical efficiency of the generator η as function of the load factor *K* for some values of *Ha*.

In fig. (7) we plot the electrical efficiency of the generator, given in eq. (35), as a function of *K* for several *Ha*. We observe in the graph that the electric efficiency of the generator has maximum values in the case of short circuit operation. Also, we observe that efficiency for open circuit conditions is null. For all other cases, we observe that a better efficiency can be obtained by increasing *Ha*. In general, the maximum efficiency is reached as *Ha* tends to infinity. In addition, note that the maximum electrical power does not coincide whit the maximum efficiency of the generator.

Conclusion

We have analyzed the performance of a direct current MHD generator in annular geometry. The MHD equations were solved under the inductionless approximation and assuming a very small annular space of the duct. We found that the internal resistance of the generator depends on the Hartmann

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number, *Ha*, and the internal radius of the duct. As for the electrical performance, the maximum electrical power delivered by the generator takes place in the condition where the internal resistance equals the load resistance, as occurs in the Faraday generator. The analyzed generator has characteristics of the Faraday generator and the Hartmann flow, although the Hartmann layers are inexistent due to magnetic field configuration.

Appendix A. Formulation of the small gap approximation.

It is useful to express the momentum equation (10), the Ohm´s law (3) and the charge conservation condition, $j_r \sim 1/r$, in terms of the reduced variable $\chi=(r-r_i)/\delta$, where $\chi \in [0,1]$. The respective resulting equations are

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{v}{\delta^2}\left(\frac{\partial^2 u_z}{\partial \chi^2}+\frac{1}{\left(\chi+\frac{r_i}{\delta}\right)}\frac{\partial u_z}{\partial \chi}\right)-\frac{\sigma c}{\rho}\frac{S}{\delta^2\left(\chi+\frac{r_i}{\delta}\right)^2}=0 \quad (36)$$

$$j_r=-\sigma\left(\frac{1}{\delta}\frac{\partial \varphi}{\partial \chi}+\frac{u_z S}{\delta\left(\chi+\frac{r_i}{\delta}\right)}\right) \quad (37)$$

And

$$j_r=\frac{-\sigma c}{\delta\left(\chi+\frac{r_i}{\delta}\right)} \quad (38)$$

Now, we follow the formulation given by Rudraiah (1966) and Ávalos *et al*. (2014), where the annular, $\delta=r_0-r_i$, is assumed to be small enough so that $r_i/\delta \gg 1$. Considering this assumption, we develop the Taylor expansion until the first order term of $(\chi+r_i/\delta)^{-1}=\delta/r_i+O(2)$ and $(\chi+r_i/\delta)^{-2}=(\delta/r_i)^2+O(2)$. These results are applied to equations (36), (37) and (38) leading to equations (12), (13) and $j_r \approx (-\sigma c)/r_i$ as is shown in Section 3.3.

With this approach, the Lorentz force becomes constant and the effect of curvature in viscous terms of momentum equation is just negligible.

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