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# Theoretical analysis of two stage shock isolation mount with switchable stiffness control

## Análisis teórico de un soporte de dos etapas para aislamiento de impacto con control de rigidez conmutable

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### Abstract

Two-stage vibration mounts are a two degree of freedom system that can improve high-frequency isolation at the cost of added mass. However, the shock response and isolation characteristics of these mounts have not been previously studied. This paper presents a theoretical study of a two-stage mount under shock excitation. The shock response is evaluated in terms of the mass ratio between the two stages of the mount. It is demonstrated that improved shock isolation can be achieved depending on the value of the secondary mass. Furthermore, a variable stiffness strategy is applied considering an on-off switching logic between a high and a low stiffness value during residual vibration. The effects of using the proposed strategy are investigated when applied in the first, second, and both isolation stages. It is found that a passive two stage mount can improve shock isolation and that the use of switching stiffness further improves the isolation performance, particularly in the residual stage where it acts as added damping. The main contribution of this work is to propose a novel semi-active isolation model based on two existing approaches that can lead to better shock isolation.

### Resumen

Los soportes antivibratorios de dos etapas son sistemas de dos grados de libertad que pueden mejorar el aislamiento a altas frecuencias con el costo de incrementar la masa. Sin embargo, las características de respuesta de impacto y aislamiento de estos soportes no han sido previamente estudiadas. Este artículo presenta un estudio teórico de un soporte de dos etapas bajo excitación de impacto. La respuesta de impacto se evalúa en términos de la razón de masas entre las dos etapas del soporte. Se demuestra que se puede conseguir un aislamiento de impacto mejorado dependiendo del valor de la masa secundaria. Además, se aplica una estrategia de variación de la rigidez considerando una lógica de apagado encendido entre valores de alta y baja rigidez durante la etapa de vibración residual. Se investigan los efectos de la estrategia propuesta cuando es aplicada en la primera, segunda y ambas etapas de aislamiento. Se encuentra que el uso de un soporte pasivo de dos etapas puede mejorar el aislamiento de impactos y que el uso de rigidez conmutable mejora aún mas el rendimiento de aislamiento, particularmente en la etapa de vibración residual en la que actúa como amortiguamiento agregado. La principal contribución de este trabajo es proponer una estrategia de aislamiento semi-activa novedosa basada en dos enfoques previamente existentes que puede resultar en un mejor aislamiento de impacto.

**Keywords:**
Variable stiffness, shock, shock isolation, two-stage mount, semiactive vibration control

**Palabras clave:**
Rigidez variable, impacto, aislamiento de impacto, soporte de dos etapas, control de vibraciones semiactivo

### Introduction

Vibration and shock isolation is usually achieved by the use of flexible supports and/or increasing the effective system mass in order to reduce the natural frequency of the isolated item. In the particular case of shock isolation, its transient nature poses a further challenge due to the short duration and usually high amplitudes involved. Nonlinearities can also occur due to the high deformations in the elastic element. Ideally, a shock isolation mount should have a low stiffness during the shock, but must be stiff enough to support elastic loads when no excitation is present and preferably lightly damped, although damping is necessary to quickly dissipate residual free vibration, as presented by Nelson [1]. The majority of the recent efforts for shock isolation consider different stiffness strategies. The use of nonlinear cubic stiffness, i.e. Duffing isolator has been recently explored theoretically by Tang and Brennan [2], and also Liu et al. [3]. These studies demonstrated that the use of cubic nonlinearities in the stiffness is advantageous for reducing absolute acceleration and displacement responses in shock excitation with a detrimental effect on relative motion. This concept has also been validated experimentally by Ledezma-Ramirez et al. [4] using a mechanically suspended permanent magnet located between two electromagnets. By varying the intensity and polarity of the voltage supplied a nonlinear stiffness effect is achieved.

In vibration control there is a tradeoff between isolation during resonances and in higher frequencies. Damping is required to reduce resonant amplitudes but large amounts of damping increase vibration transmission at high frequencies. Usually metal helical springs have low damping and are used as vibration isolators. In contrast, elastomer mounts such as neoprene or rubber have viscoelastic effects, and even though they have considerable damping, their behavior at high frequency is better compared with viscous models, i.e. approximately 40 dB/decade vs 20 dB/decade of a viscously damped system. For particular applications, especially those concerning high frequency vibration and shock, it is important to have a low transmission at high frequencies. Another way to improve vibration isolation at a higher frequency a two-stage mount, which is a two degree of freedom model can be used. In order to improve vibration isolation at a higher frequency a two-stage mount, which is a two degree of freedom model can be used. When using this intermediate mass system, high-frequency transmissibility rolls off at 80 dB/decade compared to 40 dB/decade for an undamped single degree of freedom system. However, the cost of adding this second stage is evident as the total mass of the system increases as well as the space required, plus the addition of a second resonance which justifies its use only when high-frequency vibration is a concern. Although two-stage mounts are well documented and have been applied in practice for many years i.e. Rivin [5], their shock isolation properties and response are not well documented. Snowdon and Parfitt [6] studied the response of the two-stage mount under an acceleration step, finding that a large secondary mass leads to a reduced acceleration, while damping does not affect the response as it is on an equally damped simple mount. To the knowledge of the authors, this is the only study concerned with shock isolation considering a linear two-stage mount, and it is limited only to a step excitation and uniform damping. In 1999 Shekhar et al. [7] revisited the two-stage mount considering linear spring elements and nonlinear cubic damping finding that nonlinear damping in the primary system leads to better isolation in terms of absolute acceleration, displacement, and relative motion. In contrast, the response to harmonic inputs considering nonlinear elements in the two-stage mount has been considered in different aspects. For example, Zhu et al. [8] studied a combination of quadratic damping and cubic stiffness, finding that vibration amplitude can be obtained by adjusting the secondary nonlinear stiffness and damping. Gatti et al. [9] investigated the analytical response of a two degree of freedom system where the main stage is a linear oscillator coupled with a quasi-zero stiffness stage. Lu et al. [10] analyzed a two-stage isolator with hardening nonlinearity and viscous damping. The previous studies found that force and displacement transmissibility are improved in the nonlinear isolator compared to the linear system. Furthermore, Lu et al. [11] also investigated the effect of nonlinearity in the first, second, and both stages, finding that the best combination in terms of isolation is to have a nonlinearity in the second stage with high damping, while the first stage is linear and lightly damped. Experimental work carried by Lu et al. [12] demonstrated these later theoretical studies using bi-stable plates finding an improvement in displacement transmissibility compared with the linear isolator. Wang et al. [13] also found that high damping in the nonlinear second stage and a large intermediate mass attached to a soft spring in the first linear stage is beneficial. A recent contribution by Barbieri et al [14] proposed an active model for seismic vibration control considering a two-stage system. In this model the stiffness of the first stage, i.e. the one connected to the base, its switched between a low and a high value depending upon the velocity of the main mass, while the second stiffness is kept constant, showing that the strategy is effective in isolating vertical motions. As can be seen from the revised literature, the shock isolation properties of the two-stage mount have not been properly documented. As a result, this paper aims to contribute with an analysis of the shock response of the two-stage mount, compared with the classical SDOF model. Furthermore, a switching stiffness strategy that has been successful for SDOF systems is applied to the two-stage mount. The result is a novel proposal that might help to provide enhanced shock isolation. The paper is structured as follows. The response of a SODF to a versed sine pulse is briefly revised for later comparison. Then, the two-stage mount is introduced and its response is presented as a function of the input duration and mass ratio. The switchable stiffness strategy is then implemented in the first, second, and both stages of isolation, analyzing and discussing the response. The several situations described are presented in Figure 1, which depicts the schematic models for several isolators. In this work, the properties of the different approaches are discussed highlighting advantages and disadvantages. The paper ends with concluding remarks and suggestions for further work. The main contributions of this work are, first to present the shock response of a well-known passive isolation system, i.e. the two-stage model, which has not been documented before, and then to combine the two existing isolation approaches, one passive and the other semi-active, in order to propose a further isolation strategy that can lead to improved shock isolation.

## Background

The standard approach for predicting the shock response of a linear system is to consider a pulse function as external excitation, such as a half sine, versed sine, rectangular, triangular, or another pulse function and evaluate a particular response parameter as a function of the relative duration of the shock. The duration of the pulse and the natural period of the system are related by the period ratio $\frac{\tau}{T}$ where τ is defined as the shock duration and $T$ is the natural period of the system. The equation of motion of system S1, when subjected to a versed sine acceleration pulse of maximum amplitude $\ddot{y}(t)=\frac{1}{2}\left(1-\ddot{y}_p\cos\frac{2\pi t}{\tau}\right)$, is:

$$\ddot{z}+2\zeta\omega_n\dot{z}+\omega_n^2 z=-\ddot{y} \tag{1}$$

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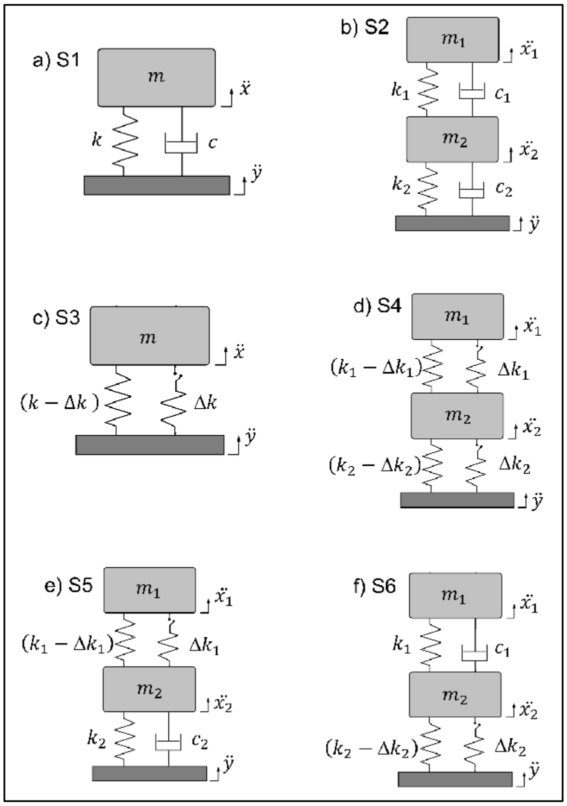


**Figure 1**. Isolation systems studied (a) S1 Single mount system, (b) S2 two-stage mount system, (c) S3 single mount with switchable stiffness, (d) S4 two-stage system with switching stiffness on both stages, (e) S5 two-stage system with switching on the main stage only, (f) S6 two-stage system with switching on secondary stage only.

Where $z=x-y$ and represents the relative motion between the base displacement y and the mass displacement $x$. Figure 2(a) defines several pulses of different duration. The SDOF model S1 is under a versed sine excitation in the form of an acceleration versed sine input. The choice of this input is due to the smooth edges at the beginning and end of the shock to avoid discontinuities in higher order derivatives. The response of the system is usually separated into two stages, namely the forced response during the input, and the subsequent free residual vibration. The Shock Response Spectra (SRS) is a representation of the non-dimensional response, as a function of the period ratio $\frac{\tau}{T}$. This tool is widely used to select and design shock isolators, asses the severity of shocks and perform shock testing. Based on the relative duration of the pulse there are three typical zones in the SRS. When the pulse is short compared to the natural period the response is smaller than the input amplitude resulting in isolation from the impact. Amplification occurs when the pulses duration is similar to the natural period. For longer duration pulses the excitation is applied very slowly, resulting in a quasistatic response that follows closely the input. The described situations can be observed in Figure 2(b) which shows an SRS corresponding to a lightly damped SDOF system under a half sine pulse excitation. The continuous line represents the maximum response at any time, sometimes called Maximax. The relative and residual responses are given by the broken line, and the dotted line, respectively.

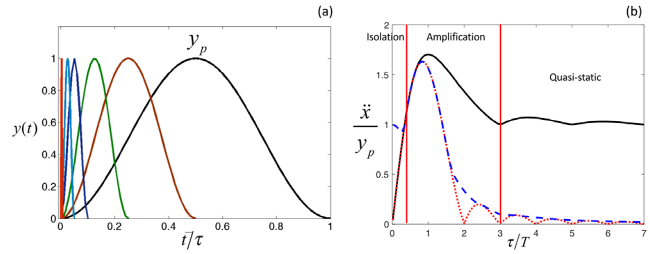


**Figure 2**. (a) Versed sine pulse for different durations. (b) Shock response spectra of an SDOF system under a versed sine pulse. (— Absolute response, - - - Relative response, · · · Residual response).

## Shock response of linear two-stage isolator

### *Mathematical model*

Consider a two-stage system S2 as depicted in Figure 1(b). The mass ratio $\mu=\frac{m_2}{m_1}$ is defined as a relationship between the additional stage $m_2$ and the isolated mass $m_1$.

$$m_1\ddot{z}_1+(c_1+c_2)\dot{z}_1-c_2\dot{z}_2+(k_1+k_2)z_1-k_2z_2=-m_1\ddot{y} \quad (2)$$

$$m_2\ddot{z}_2-c_2(\dot{z}_2-\dot{z}_1)+k_2(z_2-z_1)=-m_2\ddot{y} \quad (3)$$

Where $z_1$, $z_2$ represent the relative motions defined by $z_1=x_1-y$, $z_2=x_2-y$

Introducing the terms,

$$\omega_a=\sqrt{\frac{k_2}{m_1}},\ \omega_1=\sqrt{\frac{k_1}{m_1}},\ \omega_2=\sqrt{\frac{k_2}{m_2}},\ \zeta_1=\frac{c_1}{2m_1\omega_2},\ \zeta_2=\frac{c_2}{2m_2\omega_2},\ \mu=\frac{m_2}{m_1}$$

the equation can be expressed in non-dimensional form:

$$\ddot{z}_1+2\zeta_1\omega_2\dot{z}_1+2\zeta_2\omega_2\mu(\dot{z}_1-\dot{z}_2)+\omega_1^2z_1+\omega_a^2(z_1+z_2)=-\ddot{y} \quad (4)$$

$$\ddot{z}_2+2\zeta_2\omega_2(\dot{z}_2-\dot{z}_1)+\omega_2^2(z_2+z_1)=-\ddot{y} \quad (5)$$

The function $\ddot{y}=\ddot{y}_p\sin\frac{\pi t}{\tau}$ represents the input base acceleration in the form of a half sine pulse. The response of the system is analyzed in two stages, namely the impulsive forced vibration during the application of the pulse, and the subsequent free vibration. Numerical analysis is performed in MATLAB in order to find absolute and relative motion and acceleration. The following sections present these results in the form of shock response spectra and time histories.

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***Effect of mass ratio***

Figure 3 shows the shock response spectra for absolute acceleration and relative motion, comparing the isolation properties of the single mount S1 and the two-stage mount S2. The figures consider the cases when μ=0.2, μ=1, μ=5, μ=10 for the subplots (a), (b), (c) and (d). For all situations, damping is considered to be light and proportional, i.e. 5% of the critical value for both stages. When the secondary stage mass is a fraction of the main mass i.e. 20% in the example considered, a small reduction in absolute response is observed for very short pulses, up to the maximum response zone, i.e. around $\frac{\tau}{T}=1$. Although not included here, further reducing the secondary mass does not lead to a significant isolation enhancement, probably because when the secondary mass is small, the S2 system approaches an S1 system with two stiffness elements in series. Following with the analysis it is observed that when the secondary mass is equal to the main mass, there is no actual advantage in the isolation performance.

In addition, the response of the main mass in the S2 mount is even increased in the amplification region of the SRS when compared with the single mount S1. The effect is evident in both absolute and relative responses. For very short pulses, the response on both systems is the same. An interesting effect is the appearing of two peaks in the amplification region, due to the additional degree of freedom which is analogous to the two resonance peaks observed in the frequency response of a two degree of freedom isolator. As the secondary mass increases, the advantage of the two-stage mount S2 becomes more important particularly for μ=5, and μ=10. The main effect observed is a shift of the response towards the quasi-static region resulting in an extended isolation area, which is significantly enhanced compared with the single stage mount S1. In general terms for a larger secondary mass, the isolation area increases. However, it is important to remark that after the isolation stops, the response of the system is amplified in a broader range compared to the single mount S1. This increase also affects the relative and residual response, which are higher compared to the response of the single mount. It is interesting to note the behavior of the relative motion of system S2, which is increased for short pulses compared to the single mount S1. However, this means that the shock is being isolated by the secondary system which exhibits a large relative motion similar to the single stage mount in the isolation region, i.e. it approaches a unit value as is very close to the input amplitude. Comparable to the single mount, the cost for isolating the absolute response is a large relative motion, i.e. deflection in the elastic elements. As a result, the advantages of the two-stage mount are justifiable only if a large mass is allowed thus increasing the size of the mount, and especially for short impacts as their frequency content is higher. The benefits are then similar to those observed in harmonic excitations where the high-frequency isolation performance is increased when using a two-stage mount. Example time histories are presented in Figures 4(a) and 4(b) for period ratios of $\frac{\tau}{T}=0.25$ and $\frac{\tau}{T}=1$ respectively and a mass ratio μ=5. These plots depict the absolute and relative motion of the two stages compared to the

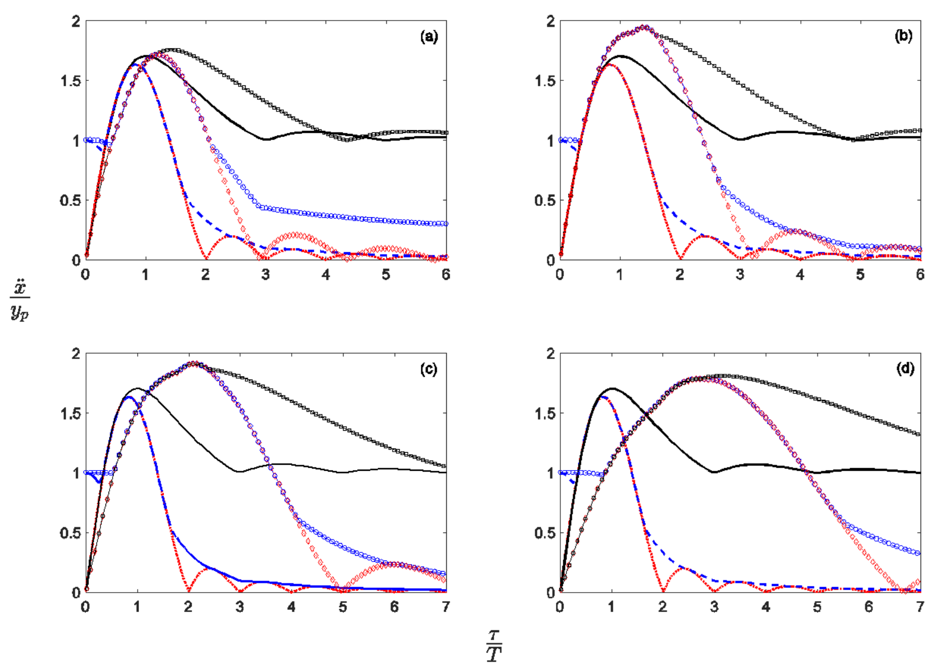


**Figure 3**. Shock response spectra for the two-stage mount S2 compared with the single mount S1.
(a) μ=0.2, (b) μ=1, (c) μ=5, (d) μ=10
(—— Absolute response S1, - - - Relative motion S1, · · · Residual response S1, ☐Absolute response S2, **O**Relative motion S2,✧ Residual response S2).

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single mount. The reduced response for short pulses in Figure 4(a) it is evident, but also the increase of relative motion during the impact. When $\frac{\tau}{T}=1$ the response is still amplified when using the two-stage mount but is smaller compared to the single stage response.

Further insight into the effect of the mass ratio can be obtained by analyzing the response contribution from each stage in the isolation system. By performing a modal decomposition in the system the response of individual modal contributions can be obtained then added by applying the superposition principle to find the global response. This process results in two decoupled systems with natural periods $T_a$, $T_b$, each one affected by a modal component of the input force. Figure 5 presents an example for $\frac{\tau}{T}=1$ where the individual decomposed contributions are shown for different values of the mass ratio, i.e. 0.2, 1, 5 and 10. When the secondary mass is small i.e. mass ratio of 0.2 the response contribution of the first decoupled system is also small. In contrast, the contribution of the second decoupled system is large. When the masses are equal, the contribution of the second decoupled system remains almost the same as the previous case but the contribution of the first one increases, thus the global response is larger. As the mass ratio increases the contribution of the first decoupled system keeps increasing, whilst the other contribution is lower. Hence, the total response of the main stage decreases.

## Implementation of switchable stiffness

### *Description of control logic*

The switching stiffness logic presented by Ledezma-Ramirez et al. [15] works in two stages. Figure 6(a) represents the model S3 used in this strategy. The first step is to reduce the initial stiffness value to a predetermined lower value when the impact starts. The ratio of stiffness reduction is defined as $\sigma=\frac{k}{\Delta k}$. Then when the shock ends the stiffness is recovered to the initial value and is then switched between on-off states according to the control law:

$$k_{eff}=\begin{Bmatrix} k & if\ x\dot{x}\geq 0 \\ k-\Delta k & if\ x\dot{x}<0 \end{Bmatrix} \quad (6)$$

As a result, the stiffness is reduced when the absolute displacement of the system reaches a maximum depicted by points B and D on Figure 6(b) and recovered when it passes through the equilibrium position at points C and E. Areas of low and high stiffness are depicted by the continuous and broken lines respectively. The shock response spectra compared to a passive SDOF system is presented in Figure 6(c) for a stiffness reduction of 50% where the benefits on the reduction of maximum response are evident. However, the main benefit is a quick reduction of residual vibration effectively adding damping through the stiffness switching, which makes this approach a good alternative to add energy dissipation in light structures. Ledezma-Ramirez et al. [15] presented a comprehensive analysis of the response concluding that much better shock isolation and quick energy dissipation is achieved. In general, better isolation is achieved as the stiffness reduction factor is increased. The effects of a delay either in the first switching event when the shock starts, and then in the residual part were also analyzed by Ledezma-Ramirez et al. [16]. The feasibility of an experimental demonstration was also presented by Ledezma-Ramirez et al. [17]. The main limitation of this approach is that a high stiffness change is required a in a very short time. It was found in the previous references that a good balance for experimental purposes and improved isolation is to have a stiffness reduction of about 50%, thus this value is considered in the following sections. Nevertheless, it is expected to obtain better isolation for higher stiffness reductions.

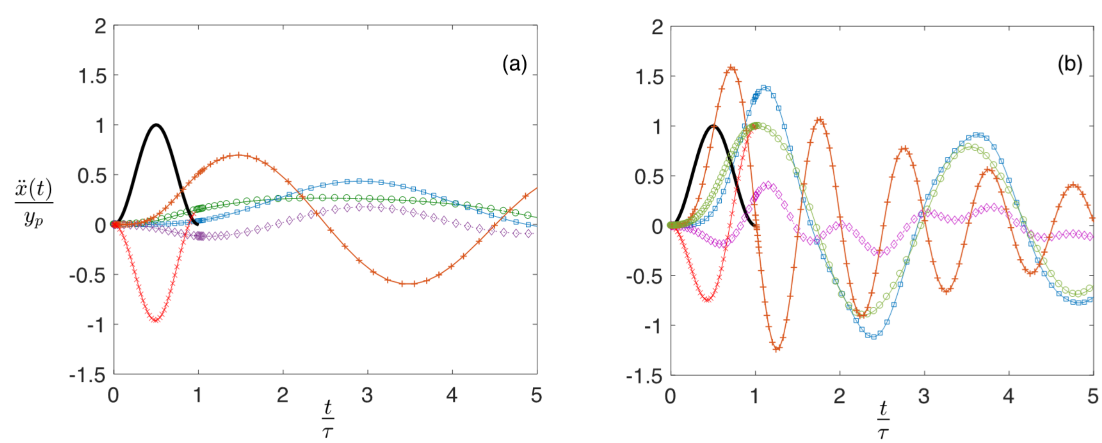


**Figure 4**. Time response for the two-stage mount under a versed sine acceleration pulse for a mass ratio μ=5.
(a) $\frac{\tau}{T}=0.25$, (b) $\frac{\tau}{T}=1$, (+ Absolute acceleration single stage, **O** Absolute acceleration response of main mass, ☐Absolute acceleration of secondary mass, ✧Relative motion between masses, ×Relative motion between the base and stage 1).

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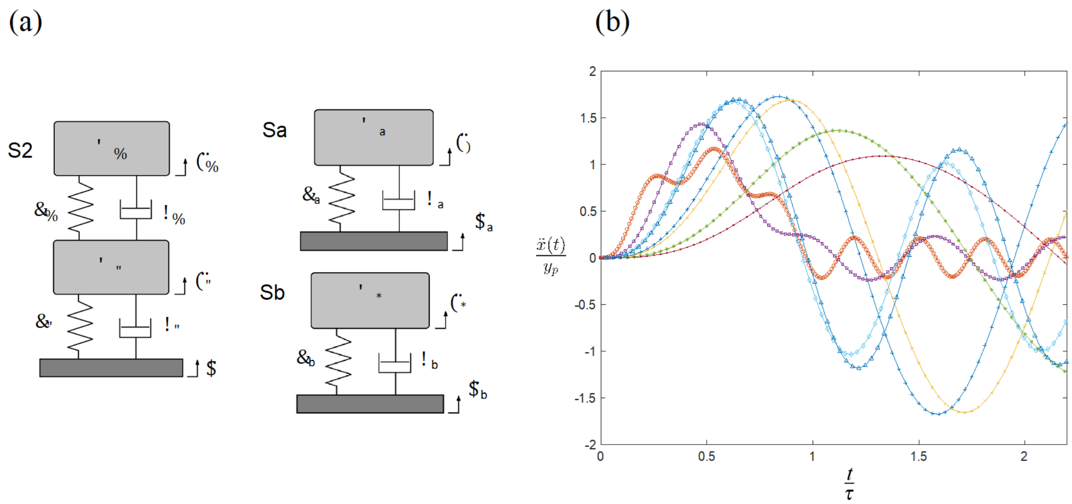


**Figure 5**. Time response of the decoupled systems comprising the two-stage mount for $\frac{\tau}{T}=1$, (+ Decoupled system Sa μ=0.2, **O** decoupled system Sb μ=0.2, × decoupled system Sa μ=1, ☐ decoupled system Sb μ=1, * decoupled system Sa μ=5, ✧ decoupled system Sb μ=5, ● decoupled system Sa μ=10, ▵ decoupled system Sb μ=10).

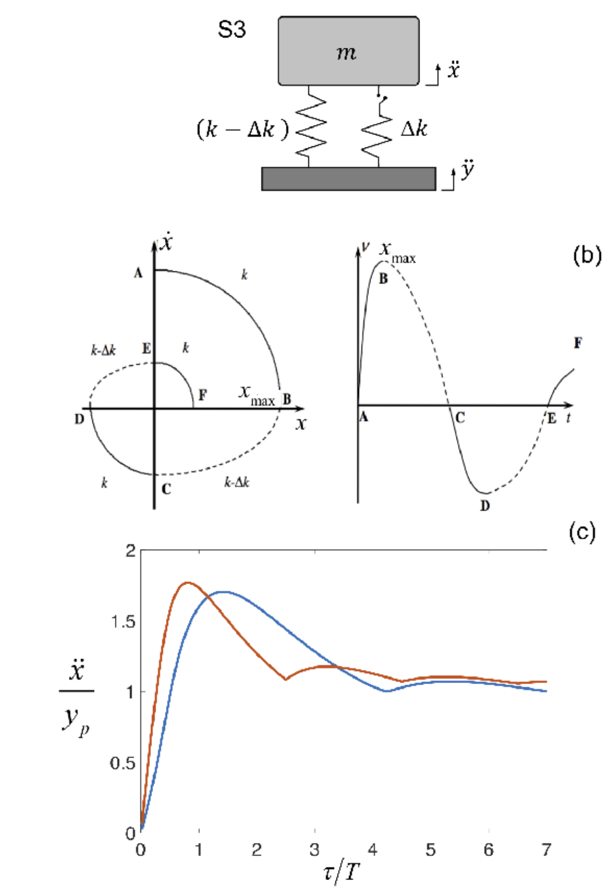


**Figure 6**. (a) Description of the switching strategy for stiffness control. (a) Schematics of the model with two springs in parallel one of them switches on and off. (b) Phase plane diagram and time history showing the switching points. (c) Shock response spectra for stiffness reduction of 50% (blue line) compared to the passive SDOF model (red line).

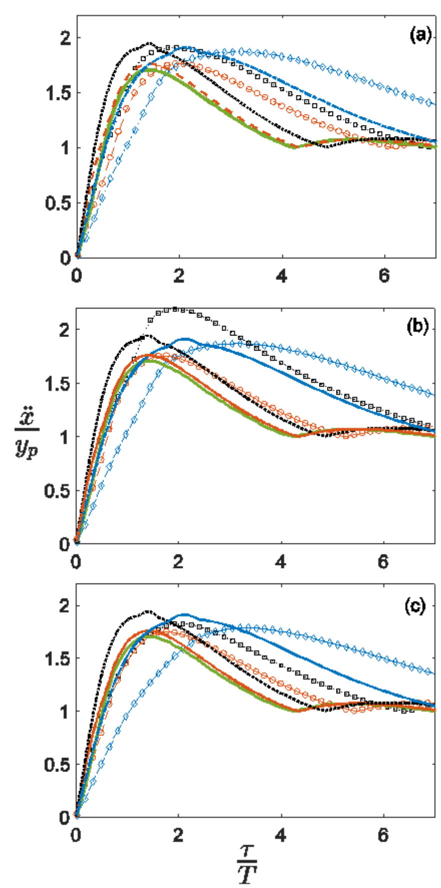


**Figure 7**. Shock response spectra for the two-stage mount with switchable stiffness. (a) Switching on both stages, (b) switching on main stage, (c) switching on secondary stage, (— Switching single mount, - - - two-stage μ=0.2, · · · two stage μ=1, - · - two stage μ=0.2, **O** two stage switching μ=0.2, ☐ two-stage switching μ=1, ✧ two-stage switching μ=5).

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**Shock response of switching system**

The models analyzed in this section are the different variants of the two-stage mount with switching stiffness, as depicted in Figure 1. The stiffness switching can be performed at both stages, i.e. system S4, only at the first stage S5, or only at the second stage S6. As a result, the switching law is applied accordingly either to the response of the main mass, the response of the secondary mass, or both. Additionally, the effect of the mass ratio can also be studied. First, the shock response spectra are presented in Figure 7. The cases presented correspond to stiffness switching on both stages, only on the main stage, and only on the secondary stages, for Figures 7(a), 7(b) and 7(c) respectively. Each case contains absolute (black lines), relative (blue lines) and residual (red lines) for three different values of the mass ratio i.e. μ=0.2, μ=1, μ=5.

In general, the trend observed is that when switching stiffness is performed on both stages, i.e. model S4 and when the mass on both stages is the same, the benefit compared to the single mount S1 is small, and the response can be even amplified. However, when the secondary mass is increased the absolute response is considerably decreased, and the isolation range is extended. When switching on both stages, it is better to have a small secondary mass (μ=0.2 in this case) as it helps in reducing the absolute response.

On the other hand, by analyzing Figure 7(b), it is clear that switching only on the main stage i.e. system S5 does not produce significant benefits compared to the equivalent passive two stage mount S2, or to the switching SDOF mount S3. Finally, Figure 7(c) shows that switching only on the secondary stage, i.e. closest to the excitation source leads to comparable benefits when switching on both stages. As before, increasing the secondary mass further improves the isolation reducing the absolute response for short to medium duration impacts and extending the isolation area. Nevertheless, for long duration pulses the response is also increased as the SRS is basically shifted towards the longer pulses region, i.e. low frequency or quasistatic impacts. The behavior observed in relative and residual responses is also similar to the passive two stage mount since the cost for isolating absolute response is an increase on relative motion. As a result, relative and residual responses are not included in order to keep the plots easy to follow. In order to gain more understanding of the behavior of the model analyzed, it is useful to observe the time histories presented in Figure 8. The representative cases selected are for a very short pulse, $\frac{\tau}{T} = 0.25$ and a pulse in the amplification region $\frac{\tau}{T} = 1$ with a mass ratio of 5 for Figures 8(a) and 8(b) respectively. Furthermore, Figures 8(c) and 8(d) are based on the same period ratios but exaggerating the value of the mass ratio to 10, in order to study a hypothetical scenario where the secondary mass is very large to apprecia-

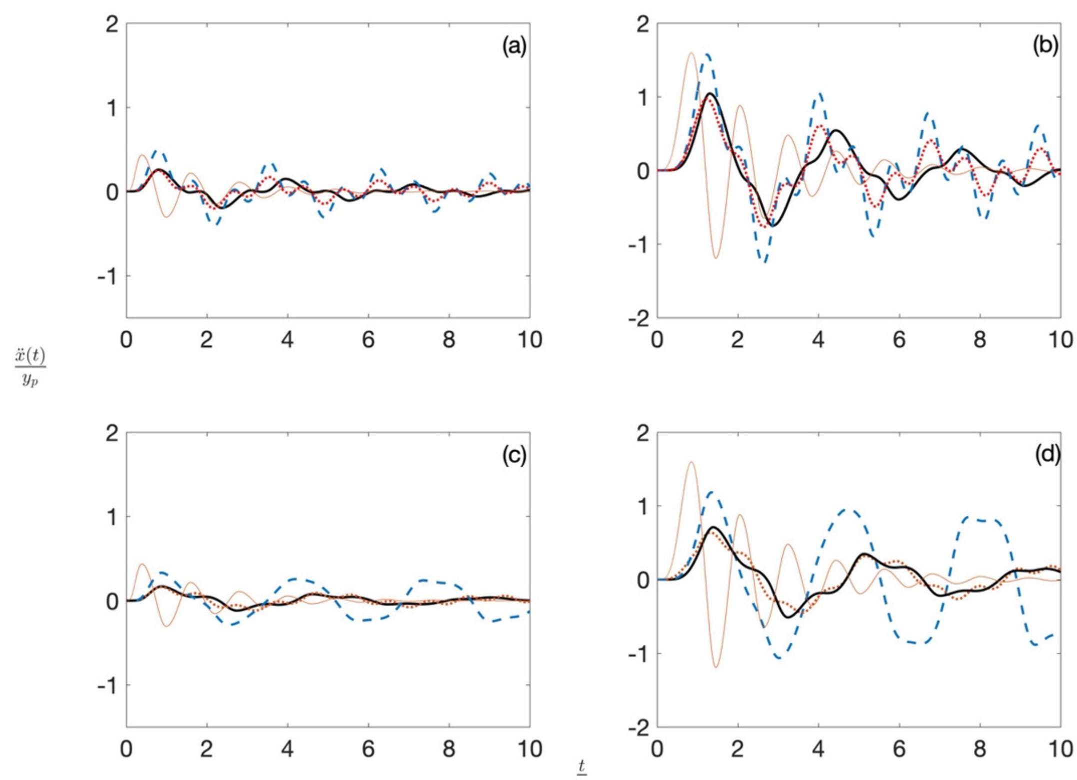


**Figure 8**. Time response of the two-stage mount with switching stiffness (thick lines) compared with the single mount (thin line) considering a 50% stiffness reduction. (a) μ=5, $\frac{\tau}{T} = 0.25$, (b) μ=5, $\frac{\tau}{T} = 1$, (c) μ=10, $\frac{\tau}{T} = 0.25$, (d) μ=10, $\frac{\tau}{T} = 1$
(— switching on both stages, - - - Switching on main stage, · · · Switching on secondary stage).

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te the effect. Contrary to the SRS, where only the maximum response value is shown, the time histories presented depict how the response evolves. It is clear that the maximum value of the response is smaller when a large secondary mass is considered and switching on both strategies is implemented when compared to the switching SDOF mount S3. However, the residual vibration decay occurs at a much faster rate in the SDOF mount. The two-stage mount takes longer times to return to equilibrium, thus reducing the added damping. This could occur due to the two degrees of freedom being coupled, so there is energy flow between the DOF. As explained before, switching only on the second stage results in a similar degree of maximum response reduction but the time required for the vibrations to decay further increases due to the main stage being undamped. This effect is even more prominent when switching only on the main stage as the benefits on reducing maximum response are minimal resulting in a longer time to return to equilibrium.

## Discussion

A final comparison is presented in Figure 9 where the maximum absolute response of the different systems studied is compared to the reference systems and presented as a function of the period ratio. Figure 9(a) compares the response of the two-stage mount S2 with different mass ratios and the single mount S1. When the mass ratio μ is 0.2 (continuous line), an effective reduction of about 28% when the period ratio is $\frac{\tau}{T} = 0.25$ is achieved, and smaller improvements up to a period ratio of 1.2 are observed. After this point, the response is increased compared to the single mount. When the masses on both stages are equal, no benefits are obtained, as the response is equal or higher compared to the single mount. Further increasing the secondary mass results in maximum benefits of about 34% and 58% for mass ratios μ of 5 and 10 respectively. It is important to note that these values also occur at a lower period ratio of about 0.17, but the usable range is also extended up to a period ratio of 1.7 for the larger mass ratio. Moreover, after this period ratio, the response increase is even larger. When comparing the switching two stage mount S4 to the single stage passive mount S1, as presented in Figure 9(b) an improvement of 70% is gained for a period ratio of 0.1 switching on both stages for a mass ratio of 5. Switching only on the secondary stage produces virtually the same maximum benefit, with marginal improvements as the period ratio increases up to 1.8. Reducing the secondary mass to 0.2 of the main mass results in improvements of about 50% in a wider range of period ratios, up to 0.5 when switching on both stages. In this case, switching only on the secondary stage produces slightly smaller benefits of around 40%, also reducing the period ratio usable range. The comparison of the passive two stage mount S2 and the switching two stage mount S4 is presented in Figure 9(c). In this comparison, only a mass ratio of 5 is presented, as this is the best scenario and other mass ratios follow the previously presented trends. Improvements of about 70% are achieved when switching in the two stages, with similar results when switching only on the secondary stage. Finally, the comparison between the two-stage switching S4 and the single stage switching S3 is presented in Figure 9(d).

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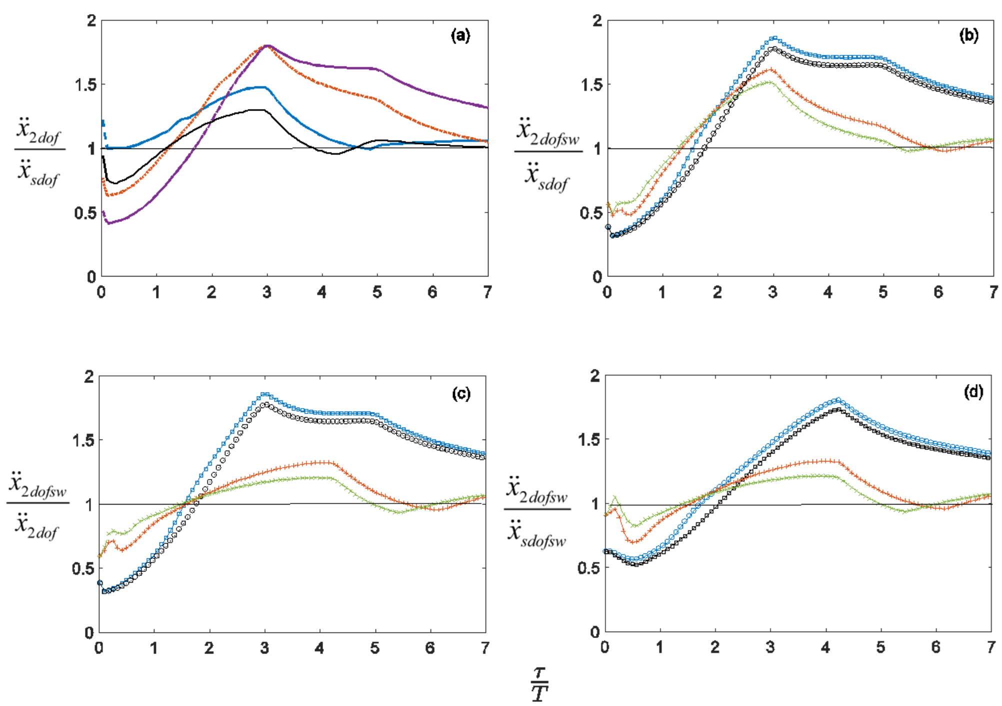


**Figure 9**. Comparison of the different strategies presented, (a) two-stage mount vs single stage mount, (b) two-stage switching vs single stage mount, (c) two-stage switching vs two stage passive, (d) two-stage switching vs single mount switching (— μ=0.2, - - - μ=1, · · · μ=5, - · - μ=10, × switching on both stages μ=0.2, □ switching on both stages μ=5, + switching on second stage μ=0.2, **O** switching on second stage μ=5).

When the mass ratio is 5, switching on both stages and on the second stage produce similar results, with a maximum improvement of about 50% and extending the useful range up to a period ratio of 2. On the other hand, for a mass ratio of 0.2, the improvement is 30% when switching on both stages and 18% when switching only on the secondary stage. From the analysis presented, two main conclusions can be drawn. If a passive mount is implemented, using the second stage with a small secondary mass can lead to important response reductions for short pulses, while increasing the mass further improves the response and extends the usable range.

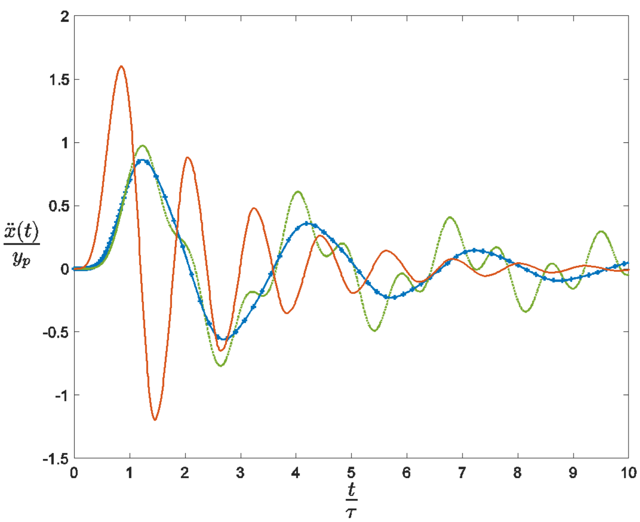


Figure 10. Time response of a two-stage mount with μ=5, $\frac{\tau}{T}=1$ considering stiffness switching on the second stage and viscous damping in the main stage. (— SDOF switching, + Switching on the second stage with damping in the main stage, · · · Switching on the second stage without damping).

When implementing a semi-active strategy, adding a second stage with switchable stiffness leads to much higher reductions compared to the other systems considered. Switching on both stages does not produce significant improvements. If space and weight is a concern a small secondary stage will produce acceptable improvements, while increasing the mass yields to significant better isolation. In order to improve residual vibration decay it is suggested to add passive damping on the main stage while switching the stiffness only on the secondary stage as depicted in Figure 10. This plot compares the response for a shock pulse resulting in a period ratio of 1, and a mass ratio of 5. It is clear that switching only on the second stage with 10% of the critical damping on the main stage leads to a much rapid residual vibration suppression when having an undamped main stage.

## Conclusions

This paper presented a theoretical study of the shock response and isolation of several two stage isolation mounts, using passive and semiactive approaches. First, a passive two stage mount was analyzed and compared with the single mount examining the effect of the value of the secondary mass. It was found that adding a large second stage compared to the main mass to isolate helps in reducing the absolute shock response with the cost of increased mass and space. The addition of switchable stiffness further improves the shock isolation properties. The effect of switching stiffness was analyzed in both stages, and on separate stages. The best compromise for shock isolation was found when applying switchable stiffness only on the secondary stage, whilst the main stage is passive and damped. The results presented show the feasibility of incorporating semi-active strategies in compound shock isolation mounts to improve shock isolation, and the experimental validation is recommended for further work. The main challenge for further work is to implement an experimentally feasible two DOF model with switchable stiffness and the corresponding control circuit. Such a model could be realized by means of smart materials, i.e. magnetorheological elastomers or piezo actuators.

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